Math 62: GC - 2nd

Solving equations graphically using GC.

\+ Identities and Contradictions

Math 72: 3.1

Solving equations graphically using GC.

and

Classifying equations as identities, contradictions or conditional.

BACKGROUND: Solving Linear Equations containing Fractions

<u>Key concept: Clearing Fractions</u>.
When there are fractions in an equation, we can choose to work with the fractions (difficult) or we can multiply both sides of the = sign by a *useful number* which makes the fractions reduce to denominators of 1, to "clear" or "cancel" the fractions.
The most efficient *useful number* is the LCD (Lowest Common Denominator) for all the fractions in the equation, but this is not the only valid method.

1) Identify all the denominators, then find the LCD for all the denominators.
2) Identify the terms (separated by add or subtract, do not include + or – signs inside parentheses) and multiply each term by the LCD.
3) Reduce the fractions formed by the LCD and the original denominator.
4) If there are any parentheses, simplify by distributing.
5) Combine any like terms on the LHS of the = sign, and combine any like terms on the RHS.
6) Collect all the variable terms on one side of the = sign.
7) Collect all the constant terms on the other side of the = sign.
8) Isolate the variable.
9) Reduce any fraction final answers.

Solve.

1) $\frac{x+7}{3} = 5$

2) $\frac{x+7}{3} + 2 = 5$

3) $\frac{2(x+7)}{3} = 5$

4) $\frac{x+7}{3} = \frac{1}{5} + x$

5) $\frac{2(x+7)}{3} = 5(x+1) + \frac{1}{5}$

Math 45 — More Examples of Clearing Fractions, Distributing, and Canceling

Solve.

① $\frac{x+7}{3} = 5$

$\cancel{3}\left(\frac{x+7}{\cancel{3}}\right) = 3 \cdot 5$ — mult both sides by 3 to clear fractions

$x+7 = 15$ — cancel on LHS, simplify on RHS

$\boxed{x = 8}$ — subtract 7

② $\frac{x+7}{3} + 2 = 5$

Method 1: Subtract 2 first

$\frac{x+7}{3} = 3$ — subtract 2 from both sides

$\cancel{3}\left(\frac{x+7}{\cancel{3}}\right) = 3 \cdot 3$ — mult both sides by 3 to clear fractions

$x+7 = 9$ — Cancel LHS, Simplify RHS

$\boxed{x = 2}$ — subtract 7

Method 2: Clear fractions first

$3\left[\frac{x+7}{3} + 2\right] = 3 \cdot 5$ — mult both sides by 3 to clear fracs.

$\cancel{3}\left(\frac{x+7}{\cancel{3}}\right) + 3 \cdot 2 = 15$ — distribute

$x+7+6 = 15$ — cancel

$x+13 = 15$ — combine like terms

$\boxed{x = 2}$

③ $\frac{2(x+7)}{3} = 5$

Method 1: clear fractions first

$\cancel{3} \cdot \frac{2(x+7)}{\cancel{3}} = 3 \cdot 5$ — mult both sides by 3

$2(x+7) = 15$ — cancel

$2x+14 = 15$ — distribute

$2x = 1$ — subtract 14

$\boxed{x = \frac{1}{2}}$ — divide by 2.

continued

③ Method 2: Distribute first

$\frac{2x+14}{3} = 5$ — distribute

$\cancel{3}\left(\frac{2x+14}{\cancel{3}}\right) = 3 \cdot 5$ — mult by 3 to clear fractions

$2x+14 = 15$ — cancel

$2x = 1$ — subtract 14

$\boxed{x = \frac{1}{2}}$ — divide by 2.

④ $\frac{x+7}{3} = \frac{1}{5} + x$

$\overset{5}{\cancel{15}}\left(\frac{x+7}{\cancel{3}}\right) = 15\left(\frac{1}{5} + x\right)$ — mult by 15 (CD of 3 & 5) to clear frac

$5(x+7) = \overset{3}{\cancel{15}} \cdot \frac{1}{\cancel{5}} + 15 \cdot x$ — cancel LHS, dist RHS

$5x + 35 = 3 + 15x$ — cancel RHS, dist LHS

$35 = 3 + 10x$ — subtract 5

$32 = 10x$ — subtract 3

$\frac{32}{10} = x$ — divide by 10

$\boxed{x = \frac{16}{5}}$ — reduce + flip

Basic Method:

- Find common denominator (CD)
- Multiply both sides by CD
- Distribute to each term
- Cancel only when there are denominators, else multiply
- Combine like terms on each side.
- Collect x's on one side of equation
- Collect constants on other side.
- Reduce any fraction answers.

$\boxed{\text{Math 45}}$ One more example of clearing fractions, dist, cancel.

(5) $\dfrac{2(x+7)}{3} = 5(x+1) + \dfrac{1}{5}$

Method 1: Mult by CD first:

$\cancelto{5}{15}\left\{\dfrac{2(x+7)}{\cancel{3}}\right\} = 15\left\{5(x+1) + \dfrac{1}{5}\right\}$

$5\{2(x+7)\} = 15\cdot 5(x+1) + \cancelto{3}{15}\dfrac{1}{\cancel{5}}$

$5\{2x+14\} = 75(x+1) + 3$

$$\begin{array}{r} \overset{2}{1}4 \\ \times\ 5 \\ \hline 70 \end{array}$$

$10x + 70 = 75x + 75 + 3$

$10x + 70 = 75x + 78$

$70 = 65x + 78$

$-8 = 65x$

$\boxed{\dfrac{-8}{65} = x}$

Method 2: Distribute first.

$\dfrac{2(x+7)}{3} = 5(x+1) + \dfrac{1}{5}$

$\dfrac{2x+14}{3} = 5x + 5 + \dfrac{1}{5}$

$\cancelto{5}{15}\left(\dfrac{2x+14}{\cancel{3}}\right) = 15\left(5x + 5 + \dfrac{1}{5}\right)$

$5(2x+14) = 15\cdot 5x + 15\cdot 5 + \cancelto{3}{15}\cdot\dfrac{1}{\cancel{5}}$

$10x + 70 = 75x + 75 + 3$

$10x + 70 = 75x + 78$

$-8 = 65x$

$\boxed{-8 \quad = x}$

EQUATION VERSUS EXPRESSION:

Using LCD to clear denom from equation:

Solve. $\dfrac{3x-2}{-2} = \dfrac{x-4}{-5}$ LCD $= \dfrac{-10}{1}$

$$\frac{-10}{1}\left(\frac{3x-2}{-2}\right) = \frac{-10}{1}\left(\frac{x-4}{-5}\right)$$ mult by $\dfrac{\text{LCD}}{1}$ both sides

$$\frac{-10}{-2}\left(\frac{3x-2}{1}\right) = \frac{-10}{-5}\left(\frac{x-4}{1}\right)$$ rearrange to show reduce fractions

$$5(3x-2) = 2(x-4)$$

etc...

Finding equivalent fractions with common denominator:

Simplify. $\dfrac{3x-2}{-2} + \dfrac{x-4}{-5}$ LCD $= 10$ or -10 (using -10)

$$\frac{5}{5}\frac{(3x-2)}{-2} + \frac{2}{2}\frac{(x-4)}{-5}$$ mult each fraction by 1 using missing factor

$$\frac{5(3x-2)}{-10} + \frac{2(x-4)}{-10}$$

$$\frac{15x-10}{-10} + \frac{2x-8}{-10}$$

etc...

BACKGROUND: Classifying Linear Equations as Identities or Contradictions

Key concept: Variables add to zero and disappear.
When solving any equation, sometimes the variables disappear.
This is correct work!

1) If the resulting statement is TRUE (like 0=0 or 7=7) then any real number will make the equation true. We classify the equation as IDENTITY, and if solving, we say ALL REAL NUMBERS, or $\{x|x \in R\}$.
2) If the resulting statement is FALSE (like 0=1 or 7=3) then no real number will make the equation true. We classify the equation as CONTRADICTION, and if solving, we say NO SOLUTION, or Ø.

Solve and classify.

1) $21(b-1)+3=3(7b-6)$

4) $\frac{3x+14}{2}=x-2+\frac{x+18}{2}$

2) $2(s+2)=2(s+1)+3$

5) $\frac{5(x+3)}{3}-x=\frac{2(x+8)}{3}$

3) $x+7=\frac{2x+6}{2}+4$

Math 45 More Examples on Identities & Contradictions

Solve.

① $21(b-1)+3 = 3(7b-6)$

$21b - 21 + 3 = 21b - 18$ distribute

$\underbrace{21b - 18}_{\text{same LHS as RHS !}} = 21b - 18$ combine like terms on LHS.

• Print handout for lec
✓ Submit
Scan for 3.1 lec.
M72
post

$-18 = -18$

$0 = 0$

IDENTITY - ______________ solutions

② $2(s+2) = 2(s+1)+3$

$2s+4 = 2s+2+3$ distribute

$2s+4 = 2s+5$ combine like terms on RHS

$4 = 5$ subtract $2s$ from both sides

CONTRADICTION - no solution

③ $x+7 = \frac{2x+6}{2} + 4$

$2(x+7) = 2\left[\frac{2x+6}{2} + 4\right]$ mult by 2 to clear fractions

$2x+14 = \cancel{2}\cdot\left(\frac{2x+6}{\cancel{2}}\right) + 2\cdot 4$ distribute

$2x+14 = 2x+6+8$ cancel

$2x+14 = 2x+14$ combine like terms on RHS.

$14 = 14$ subtract $2x$ from both sides

IDENTITY - all real #s are solutions

Basic Method: clear fractions
distribute
combine like terms on each side separately
subtract/combine x's
{ TRUE result = IDENTITY, all real #s
{ FALSE result = CONTRADICTION, no solution

Math 45 p 2

Practice Solving exceptional cases (Identities & Contradictions)

Solve.

(4) $\dfrac{3x+14}{2} = x - 2 + \dfrac{x+18}{2}$

$\cancel{2}\left(\dfrac{3x+14}{\cancel{2}}\right) = 2\left(x - 2 + \dfrac{x+18}{2}\right)$ clear fractions

$3x+14 = 2x - 4 + \cancel{2}\left(\dfrac{x+18}{\cancel{2}}\right)$ distribute

$3x+14 = 2x - 4 + x + 18$ cancel

$\underbrace{3x+14} = \underbrace{3x+14}$ combine like terms on RHS

LHS is same as RHS

$\boxed{\text{IDENTITY all Real \#s are solutions}}$

(5) $\dfrac{5(x+3)}{3} - x = \dfrac{2(x+8)}{3}$

$3\left(\dfrac{5(x+3)}{3} - x\right) = \cancel{3} \cdot \dfrac{2(x+8)}{\cancel{3}}$ clear fractions

$\dfrac{\cancel{3} \cdot 5(x+3)}{\cancel{3}} - 3x = 2(x+8)$ distribute + cancel

$5x + 15 - 3x = 2x + 16$ distribute

$2x + 15 = 2x + 16$ combine like terms on LHS

$15 = 16$ subtract 2x both sides. (Combine x's first)

$\boxed{\text{CONTRADICTION No solution}}$

Math 72 3.1 Identities and Contradictions

Solve each equation and classify as conditional, identity or contradiction.

(1) $2(z-1)+z = 4z+2$

$2z-2+z = 4z+2$ distribute

$3z-2 = 4z+2$ combine like terms

$-2 = z+2$ subtract $3z$ both sides

"Solve": $\boxed{-4 = z}$ subtract 2 both sides

This equation is $\boxed{\text{conditional}}$ (It is true on the condition that z is -4.)

(2) $2(z-1)+z = 3z+2$

$2z-2+z = 3z+2$

$3z-2 = 3z+2$

$-2 \neq 2$

"Solve": $\boxed{\text{no solution}}$ No value of z makes this equation true.

This equation is a $\boxed{\text{contradiction.}}$ All values of z contradict the truth.

(3) $2(z-1)+z = 3z-2$

$2z-2+z = 3z-2$

$3z-2 = 3z-2$

$-2 = -2$

"Solve": $\boxed{\text{all real numbers are solutions}}$ Any real value of z makes this equation true.

This equation is an $\boxed{\text{identity.}}$ The left side has the same value or identity as the right side.

Math 70 4.5 Approximating the Solution(s) to an Equation Using the Graphing Calculator

Intersection of Graphs Method:

1) Solve $2\pi x - 5.6 = 7(x - \pi)$ graphically using your graphing calculator.
   (a) For this equation, what function(s) do you graph in your calculator?

   (b) For the method you chose, where/how do you find the solution(s)?

   (c) Round the solution to four decimal places.

x-intercepts Method:

2) Solve $2\pi x - 5.6 = 7(x - \pi)$ graphically using your graphing calculator.
   (a) For this equation, what function(s) do you graph in your calculator?

   (b) For the method you chose, where/how do you find the solution(s)?

   (c) Round the solution to four decimal places.

Solving Linear Equations Graphically

1) Solving equations using intersection of graphs method on GC
   (GC 22) - lesson 10 4.1, 4.2, 4.3
2) Solving equations using the x-intercepts method on GC
   (GC 23)

In the real world, equations often include many complexities that make them difficult to solve (and possibly impossible to solve) by isolating the variable or other algebraic methods.

Instead, we use a calculator or computer to approximate the solutions, using either of two methods.

Example: $\underbrace{2\pi x - 5.6}_{\text{L.H.S.}} = \underbrace{7(x-\pi)}_{\text{R.H.S.}}$

L.H.S. "left hand side" R.H.S. "right-hand side"

Method 1: Intersection of graphs (GC22)

In GC: $y_1 = LHS$
$y_2 = RHS$

Solutions are where graphs intersect. → Solutions are x-coordinates (ignore y)

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = CALC Menu
5. Intersect

Method 2: x-intercepts of difference (GC 23)

$$\begin{array}{rcl} LHS & = & RHS \\ -RHS & & -RHS \\ \hline LHS - RHS & = & 0. \end{array}$$

solutions are x-coordinates (ignore y-coords)

In GC: $y_1 = LHS - RHS$

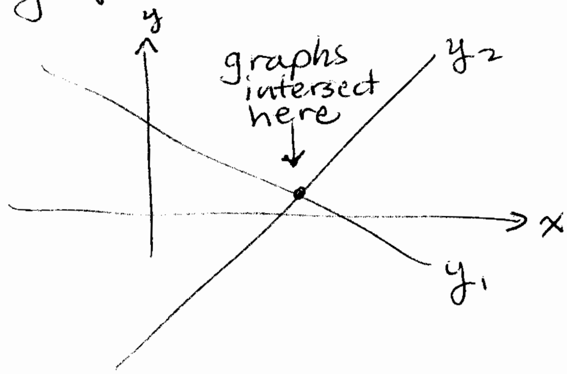
$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = CALC Menu 2. Zero

Math 70

<u>Vocabulary</u>: Intersect vs. Intercept

* These are different and not interchangeable! *

<u>to intersect</u>: This is a verb, and means that two graphs cross each other.

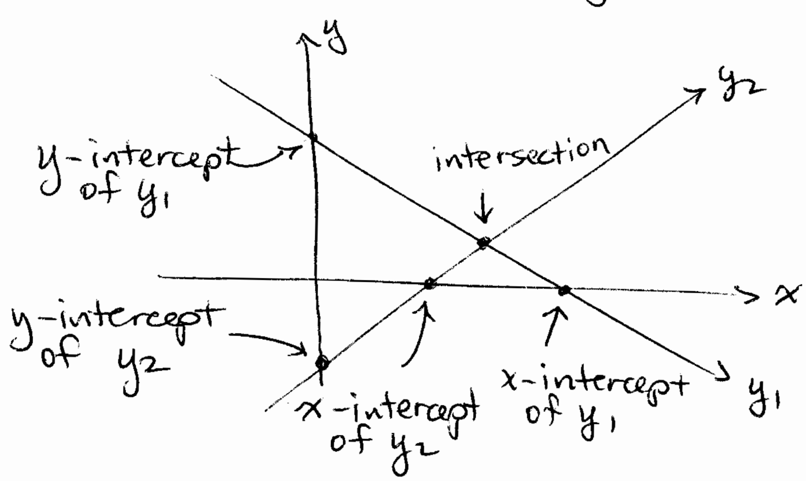


<u>intersection</u>: This is the noun that goes with "to intersect", and refers to the point where the graphs cross.

<u>intercept</u>: This is a noun in math, and must have more information to make sense.

<u>x-intercept</u>: The point where a graph crosses the x-axis.

<u>y-intercept</u>: The point where a graph crosses the y-axis.



Math 70

In an English language usage, "intercept" can be used as a verb, as in:

"The police officer intercepted the suspect three blocks later."

But we do not use "intercept" as a verb in math class.

For Spanish speakers, it's confusing because the two cognates are legitimate synonyms.

intersect = intersectar, cruzarse entrecruzar, interceptar

intercept = interceptar, atajar, atajar por en medio, cortar el paso a, cortar la retirada a, intersectar

But here are the two words that are most like English.

interceptar = intercept
intersectar = intersect

Math 70

Intersection Method:

① Solve the equation graphically. Round solution to the nearest ten-thousandth.

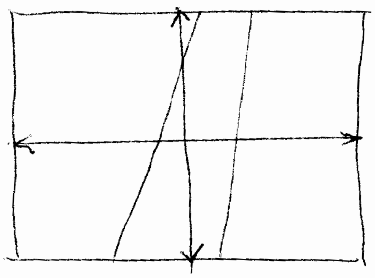
$$\underbrace{2\pi x - 5.6}_{LHS} \overset{\Uparrow}{=} \underbrace{7(x-\pi)}_{RHS}$$

In GC: $\boxed{Y=}$

$Y_1 = LHS$ $\qquad$ $Y_1 = 2\pi x - 5.6$

$Y_2 = RHS$ $\qquad$ $Y_2 = 7(x-\pi)$

Graph in a standard window:



The lines look almost parallel. But are they parallel?

$Y_1 = 2\pi x - 5.6$ has slope $m = 2\pi \approx 6.3$

$Y_2 = 7(x-\pi) = 7x - 7\pi$ has slope $m = 7$.

$6.3 \neq 7$ slopes are different.

These lines do intersect.

Where?

Somewhere above current screen $\Rightarrow$ increase YMAX.

May need to increase XMAX.

* Play with your $\boxed{\text{WINDOW}}$ settings until you can see the point of intersection.

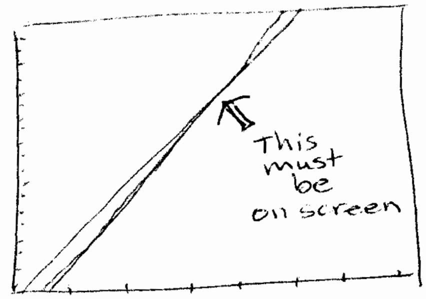
If it's not visible, your GC cannot find it.

Math 70

Intersection Method, continued.

$\boxed{\text{WINDOW}}$ for example

XMIN = 0
XMAX = 40
XSCL = 5
YMIN = 0
YMAX = 180
YSCL = 10



You MIGHT ALSO use $\boxed{\text{ZOOM}}$ OUT and then $\boxed{\text{ZOOM}}$ IN.
What window settings are used doesn't matter, only that the point of intersection must be visible.

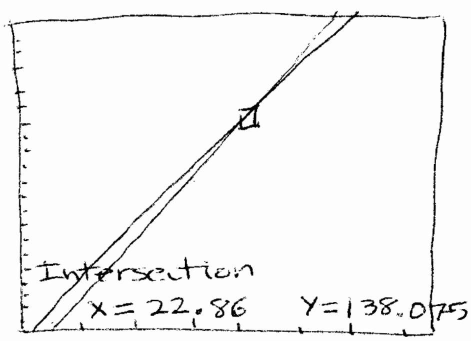
To calculate point of intersection

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = $\boxed{\text{CALC}}$

5. Intersect.

1st curve? $\boxed{\text{ENTER}}$
2nd curve? $\boxed{\text{ENTER}}$ } Y_3, Y_4 etc must be empty in your $\boxed{\text{Y=}}$ menu

Guess? (if more than one solution, use cursor) $\boxed{\text{ENTER}}$



Look at bottom of screen for answer

22.866647

Solution $\boxed{x \approx 22.8666}$

So what is the y value? It's the y-coord we get by substituting back into LHS or RHS.

$$\underbrace{2\pi x - 5.6}_{} = \underbrace{7(x - \pi)}_{}$$

ycoord 138.075 = ycoord 138.075

This is an artifact of using this method. But we are still solving for $\underline{\underline{x}}$.

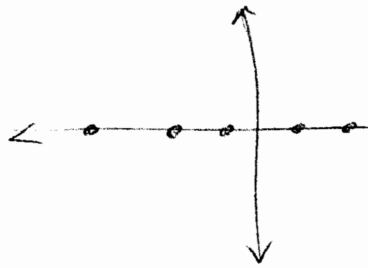
Math 70

X-intercept Method:

1) Solve the equation graphically. Round solution to the nearest ten-thousandth.

$2\pi x - 5.6 = 7(x - \pi)$

To use x-intercept method, remember:



All points on the x-axis have y-coordinate 0.

We need $y = 0$ in our equation.

Set equation $= 0$, with either the 0 on the LHS or the zero on the RHS:

$0 = 7(x-\pi) - 2\pi x + 5.6$ OR $2\pi x - 5.6 - 7(x - \pi) = 0$

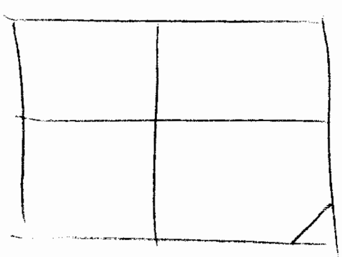
We will graph only one function. Must clear y_2.

$\boxed{y=}$

$y_1 = 7(x-\pi) - 2\pi x + 5.6$

$y_2 = \boxed{\text{CLEAR}}$

$\boxed{\text{ZOOM}}\ \boxed{6}$

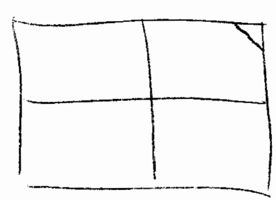


$\boxed{y=}$

$y_1 = 2\pi x - 5.6 - 7(x - \pi)$

$y_2 = \boxed{\text{CLEAR}}$

$\boxed{\text{ZOOM}}\ \boxed{6}$

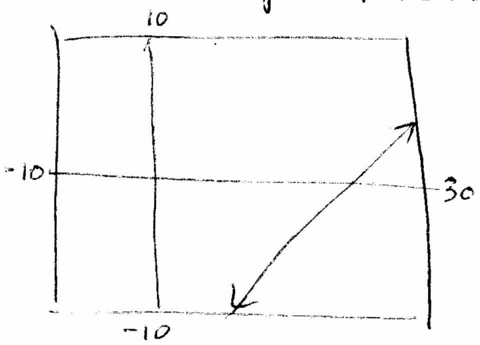


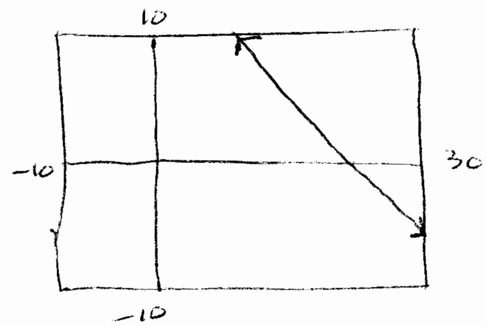
Adjust the window so that the x-intercept is visible ⇒ increase XMAX

$\boxed{\text{WINDOW}}$ XMAX = 30 (for example)

Math 70

X-intercept Method, con't.





To calculate x-intercept:

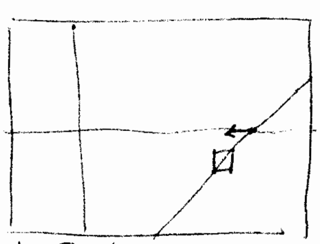
$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = $\boxed{\text{CALC}}$

2. Zero

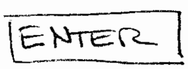
{ on 86
$\boxed{\text{GRAPH}}$ - $\boxed{\text{MORE}}$ MATH - ROOT }
F1 F1

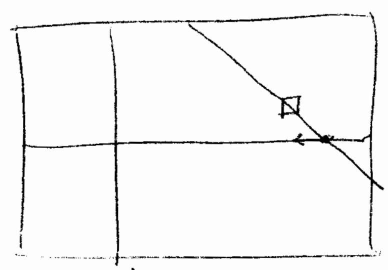
> IMPORTANT: Do NOT press $\boxed{\text{ENTER}}$ three times or the GC will give you a guaranteed error.

Left Bound? Must move cursor using ▶ or ◀ to a point on the line to the left of the x-intercept.
(You're telling the GC where to look).



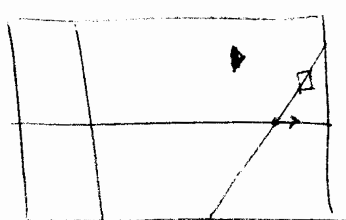
Left bound



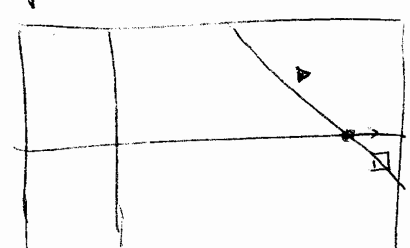


Left bound.

Right Bound? Must move cursor using ▶ to a point on the line to the right of the x-intercept.



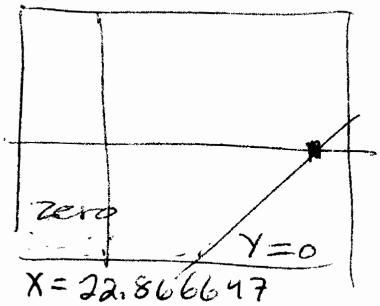
Right Bound

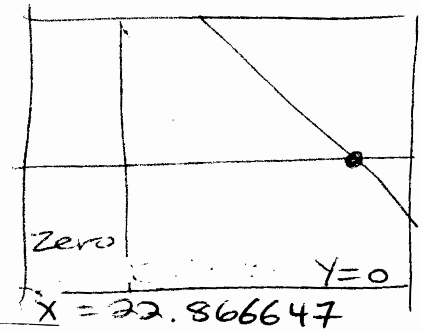


Right bound

Guess? $\boxed{\text{ENTER}}$

X-intercept method, cont.





$\boxed{x \approx 22.8666}$

Which method should you use?
Either method is valid.

	Intersection	X-intercept
Advantages	GC key strokes easier No algebra to rearrange	Easier to adjust window (only XMIN or XMAX)
Disadvantages	Harder to adjust window	Algebra at beginning to set = 0. GC keystrokes require cursor movement

Math 70 Approximating the Solution(s) to an Equation Using the Graphing Calculator

Intersection of Graphs Method:

1) Solve $2\pi x + 5.6 = 7(x - \pi)$ graphically using your graphing calculator.

(a) For this equation, what function(s) do you graph in your calculator?

$Y_1 = 2\pi x + 5.6$ ← or vice-versa

$Y_2 = 7(x - \pi)$

(b) For the method you chose, where/how do you find the solution(s)?

The x-coordinate of the point of intersection, or point where graphs intersect.

(c) Round the solution to four decimal places.

$x \approx 22.86664$

$\boxed{x \approx 22.8666}$

tenths

hundredths

thousandths

ten-thousandths

x-intercepts Method:

2) Solve $2\pi x + 5.6 = 7(x - \pi)$ graphically using your graphing calculator.

(a) For this equation, what function(s) do you graph in your calculator?

$Y_1 = 2\pi x + 5.6 - 7(x - \pi)$ ← or subtract LHS from RHS

(b) For the method you chose, where/how do you find the solution(s)?

The x-coordinate of the x-intercept or point where graph intersects the x-axis.

(c) Round the solution to four decimal places.

$x \approx 22.86664$

$\boxed{x \approx 22.8666}$